VALIDATION OF A FINITE ELEMENT CONTINUUM MODEL OF VOCAL FOLD VIBRATION

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1 Introduction

The accurate simulation of vocal fold vibration is contingent on appropriate formulation of the system's equation of motion. Lumped element and continuum formulations have been applied in previous studies to characterize this vibration under both normal and pathological conditions. The present study seeks to validate an in-house finite element code of vocal fold vibration, and to examine the resultant effects of an added unilateral sessile polyp on the fundamental frequency of vocal fold vibration. Validation is performed with a direct comparison of natural frequencies and modes of a numerical study by Luo et al. [1]. The development of the pathological vocal fold model is subsequently discussed, and trends are presented.

2 Methods

The finite element method (FEM) is applied to a discretized vocal fold system to formulate the general eigensystem given by

$$[K]\phi = \omega^2[M]\phi \tag{1}$$

where the mass matrix, $[M]$, and stiffness matrix, $[K]$, are developed using the FEM. For an N degree of freedom system, equation (1) can be solved for N pairs of eigenvectors, ϕ, and eigenvalues, ω^2 [2, p. 786]. Geometries are discretized using linear strain elements, and a combination of analytical and numerical techniques are used to evaluate the mass and stiffness contributions of each element.

3 Model Validation

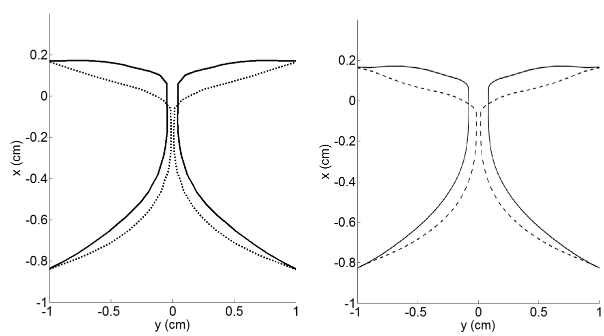
A multi-layered, three-dimensional continuum model of a healthy vocal fold is presented in [1]. This model documents the material properties, boundary conditions, and geometry of each layer in detail [1, p. 9330, 9316]. Luo et al. adopt the immersed boundary method to solve for the first four natural frequencies and mode shapes of the model, which are used in the present study for direct comparison. Convergence of these natural frequencies was observed in the FEM model for increasingly fine meshes. The results obtained from the finest mesh are used for comparison.

Table 1 contains a comparison of the first four computed natural frequencies with those reported in [1]. The maximum computed percent difference is 5 %, which corresponds with the fourth mode of vibration. Since a comparison is being drawn between two numerical studies, exact agreement would not be expected. Nonetheless, the agreement of the FEM model with the solution from [1] was deemed similar enough to conclude the computed solutions are accurate.

Table 1: Comparison of computed natural frequencies of the healthy vocal fold model

Mode number	Luo et al. (Hz)	FEM model (Hz)	Percent difference (%)
1	114	110.1	3.5
2	125	120.5	3.7
3	133	128.4	3.5
4	144	136.9	5.0

Further comparison is drawn qualitatively between the computed mode shapes. For brevity, only a comparison of the first mode is reported, in figure 1. This figure illustrates the outer periphery of the coronal cross-section, midway between the anterior and posterior vocal fold surfaces. Lateral and vertical displacements of the positive and negative eigenvectors appear to be in agreement.



(a) Computed mode shape using the FEM

(b) Computed mode shape from [1]

Figure 1: Comparison of the first mode of vocal fold vibration. Pictured is the coronal cross-section.

4 Development of a Pathological Model

Polyps are unilateral lesions which form on the cover of a vocal fold. Sessile polyps have a balloon-like appearance, and manifest as half-ellipses which protrude from the medial surface of the vocal fold [3, p. 268]. Sessile polyps have been observed between 0.3 mm to 0.7 mm in length [4, p. 129]. The material properties of polyps have a wide range of reported values, though typically, the polyp presents increased stiffness relative to the surrounding tissue.

Based on this data, a model of a vocal fold afflicted with a sessile polyp was developed, with its geometry presented in

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figure 2. Vocal fold geometry and orthotropic material properties are based on Luo et al, [1, p.9330]. The polyp is represented as a sphere whose center and diameter are treated as variable parameters. The polyp's center, c, is situated midway along the medial surface in the y direction, and may vary along the anterior-posterior direction, z, between 0.3 mm $\leq c \leq 13.7$ mm. The length, d, is bound between 0.3 mm $\leq d \leq 0.7$ mm. The polyp is treated as an isotropic material, with a fivefold increase in stiffness relative to the transverse Young's modulus of the ligament, a density of 1.1 g/cm^2, and a Poisson ratio of 0.3. Anterior, posterior, and lateral surfaces of the vocal fold are held fixed, while remaining nodes are free.

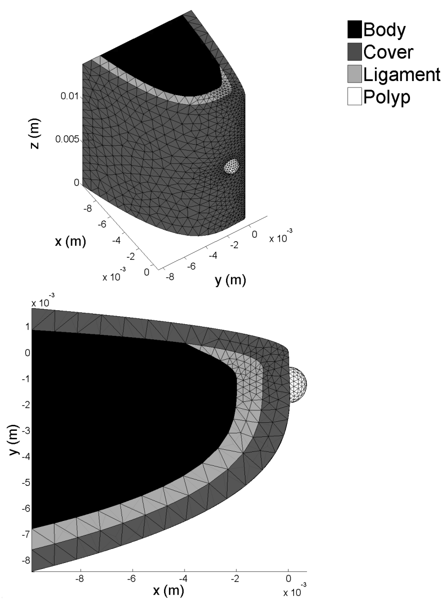

Figure 2: Pathological vocal fold mesh. The polyp position may vary along the anterior-posterior direction, z, and the polyp radius may vary from 0.3 mm to 0.7 mm.

The effects of varying c and d on the fundamental frequency are assed with two sets of simulations, similar to a study by [5]. The first set of simulations varies 0.7 mm $\leq c \leq 7$ mm at $d = 0.7$ mm for 10 trials. The second set of simulations varies 0.3 mm $\leq d \leq 0.7$ mm at $c = 7$ mm for 5 trials.

5 Results and Discussion

The effect of varying the position of the polyp along the anterior-posterior direction is shown in figure 3. As the polyp moves towards the center of the vocal fold ($c = 7$ mm), the fundamental frequency reaches a minimum. Since the vocal fold geometry is symmetric, the same trend would be expected for a set of simulations which varies 7 mm $\leq c \leq 14$ mm. For this dataset, a linear trend line was fitted. This varies from [5], which fit a quadratic curve, but otherwise, both studies present the same trend between frequency and position. Variance may be due to the smaller dataset used in [5].

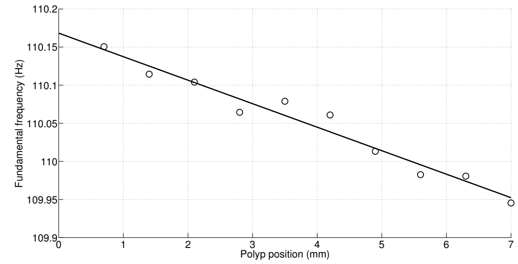

Figure 3: Effect of polyp position on fundamental frequency

The effect of varying the size of the polyp is shown in figure 4. Asymptotic behaviour is observed at the upper and lower bounds of d. Smaller polyps will have negligible influence on frequency, and inversely, larger polyps have a profound influence on frequency. This trend corresponds with studies which examine the damping of the mucosal wave on the vocal fold surface due to localized stiffness increases of the polyp.

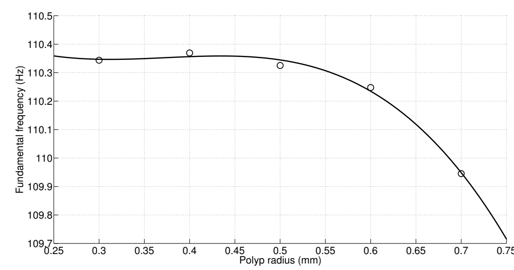

Figure 4: Effect of polyp size on fundamental frequency

The validation process of the FEM code is integral for future uses as a predictive tool. Future studies will examine mode shape variation under the influence of polyps, as well as acoustic sound radiation in a time domain analysis.

References

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